

In a football competition run over several days 18 teams are participating. Prove that at any day there are at least two teams which have completed the same number of matches.

Deadline for solution: 03/16/09.

Solution of the previous problem:

Find the product of the last two digits of the number

$$2 \cdot 2^2 \cdot 2^3 \cdot 2^4 \dots 2^{999} \cdot 2^{1,000}.$$

$$2 \cdot 2^2 \cdot 2^3 \cdot 2^4 \dots 2^{999} \cdot 2^{1,000} = 2^{1+2+3+\dots+999+1,000} = 2^{\frac{1,000 \cdot 1,001}{2}} = 2^{500,500}.$$

The last two digits of a power of 2 are following the pattern 02, 04, 08, 16, 32, 64, 28, 56, 12, 24, 48, 96, 92, 84, 68, 36, 72, 44, 88, 76, 52 and then it repeats in the same order except that the initial 02 will not come up again and the cycle starts with 04. Thus the digits repeat with a period of 20, starting with  $2^2$ . Since

$$500,499 = 20 \cdot 25,024 + 19,$$

the last two digits of  $2^{500,500}$  are 76 and so the product is 42.