Prove that is 3 divides $a^2 + b^2$ then 3 divides both a and b.

Deadline for solution: 4/12/09.

Solution of the previous problem:
What is the last digit of

$$2007^{2007^{2007}}.$$

The last digit of a number to a power is determined only by the last digit of the number. Thus the last digit of

$$2007^{2007^{2007}}$$

is the same as the last digit of

$$7^{2007^{2007}}.$$

The last digit of powers of 7 repeats with a period of 4. If the exponent is divisible by 4, the last digit is 1, if the exponent has remainder 1 when divided by 4, the last digit is 7, if the exponent has remainder 2 when divided by 4, the last digit is 9, and if the exponent has remainder 3 when divided by 4, the last digit is 3. Therefore the problem becomes: what is the remainder of

$$2007^{2007}$$

when divided by 4.

Since

$$2007 \equiv -1 \quad (mod\ 4)$$

$$2007^{2007} \equiv (-1)^{2007} = -1 \equiv 3 \quad (mod\ 4).$$

Hence the last digit is 3.