A six - digit number contains the digits 1, 2, 3, 4, 5, & 6 (each once, of course). Prove that the number cannot be a square.

Deadline for solution: 10/26/09. Send solution at gprajitu@brockport.edu or drop hard copy in Dr. Prajitura's mail box.

Last week's problem:
There are infinitely many deficient numbers having two prime factors.

Let p be a prime number, $p > 3$. The number $2p$ is deficient because its proper divisors are $1, 2, p$ and

$$1 + 2 + p < 2p \iff 3 < p.$$

The problem was solved by Elizabeth Mellen and Logan Martin.

Problem for graduate students:
Solve the equation

$$x^6 + x^5 + 4x^4 - 12x^3 + 4x^2 + x + 1 = 0.$$

Last week's problem:
For every k, there are infinitely many deficient numbers with k prime factors.

Let $p_1 < p_2 < \ldots < p_k$ be prime numbers such that $p_1 > 2k$. We will show that $p_1 p_2 \cdots p_k$ is a deficient number. This means

$$2p_1p_2\cdots p_k > (1+p_1)(1+p_2)\cdots(1+p_k) \iff 2 > \left(1+\frac{1}{p_1}\right)\left(1+\frac{1}{p_2}\right)\ldots\left(1+\frac{1}{p_k}\right)$$

This inequality is true because

$$\left(1+\frac{1}{p_1}\right)\left(1+\frac{1}{p_2}\right)\ldots\left(1+\frac{1}{p_k}\right) < \left(1+\frac{1}{2k}\right)^k < \sqrt{e} < 2.$$

There was no correct solution to this problem.