

If p, q are prime numbers such that

$$n = \frac{p^2 + q^2}{p - q}$$

is an integer, prove that $n - 1$ is a square.

Deadline for solution: 11/2/09. Send solution at gprajitu@brockport.edu or drop hard copy in Dr. Prajitura's mail box.

Last week's problem:

A six - digit number contains the digits 1, 2, 3, 4, 5, & 6 (each once, of course). Prove that the number cannot be a square.

The number is divisible by 3 but not by 9, so it cannot be a square.
The problem was solved by Elizabeth Mellen and Peter Kosek.

Problem for graduate students:

If the equation

$$x^3 + (a + 1)x + b = 0$$

has only integer solutions, none of them equal to 0, prove that $a^2 + b^2$ cannot be a prime number.

Last week's problem:

Solve the equation

$$x^6 + x^5 + 4x^4 - 12x^3 + 4x^2 + x + 1 = 0.$$

This is a reciprocal equation of degree 6. That is, the coefficient at the same distance from the end - points are equal. The method to solve an equation like this is to divide the equation by x^3 (that is, x to half of the degree) and make the substitution

$$y = x + \frac{1}{x}.$$

First, dividing by x^3 we get

$$x^3 + x^2 + 4x - 12 + 4\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} = 0$$

or

$$x^3 + \frac{1}{x^3} + x^2 + \frac{1}{x^2} + 4\left(x + \frac{1}{x}\right) - 12 = 0.$$

We know that $x + \frac{1}{x} = y$ but we also need $x^2 + \frac{1}{x^2}$ and $x^3 + \frac{1}{x^3}$.

$$y^2 = \left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$$

and so

$$x^2 + \frac{1}{x^2} = y^2 - 2.$$

$$y^3 = \left(x + \frac{1}{x}\right)^3 = x^3 + 3x + 3\frac{1}{x} + \frac{1}{x^3} = x^3 + \frac{1}{x^3} + 3y$$

and so

$$x^3 + \frac{1}{x^3} = y^3 - 3y.$$

Now we substitute in the initial equation and we get

$$y^3 - 3y + y^2 - 2 + 4y - 12 = 0$$

or

$$y^3 + y^2 + y - 14 = 0.$$

It is easy to see that 2 is a solution of the last equation and thus the equation factors as

$$(y - 2)(y^2 + 3y + 7) = 0$$

which has 3 solutions

$$y_1 = 2, \quad y_2 = \frac{-3 + i\sqrt{19}}{2}, \quad y_3 = \frac{-3 - i\sqrt{19}}{2}.$$

Now we solve three quadratic equations

$$x + \frac{1}{x} = y$$

for each of the three values we got for y .

Correct solutions were submitted by Peter Kosek, Ellen Nary and Kevin Spear.