Find

$$\lim_{x\to\infty}\left(\frac{1}{2}\left(a^{\frac{1}{x}}+b^{\frac{1}{x}}\right)\right)^x$$

where a and b are two strictly positive numbers.

Deadline for solution: 11/23/09. Send solution at gprajitu@brockport.edu or drop hard copy in Dr. Prajitura's mail box.

Last week's problem:

We assign to each side of a cube a number from 1 to 12 in a one - to - one correspondence. Prove that it is not possible to do it in such a way that the sum of the numbers assigned to the edges going into the same vertex is the same for every vertex.

Suppose that is possible. Then we get $a_1, a_2, \ldots, a_{12}$, a permutation of the numbers 1 through 12, each assigned to one of the edges, in such a way that the sum of the numbers assigned to the edges going into the same vertex is the same for every vertex. Let k be this sum. Then

$$2(a_1 + a_2 + \cdots + a_{12}) = 8k$$

because there are 8 vertices and each edge goes into two vertices.

Since the numbers are a permutation of the numbers 1 through 12 then their sum is the same as $1 + 2 + \cdots + 12 = 78$. Therefore we get

$$78 = 8k$$

which is a contradiction because 8 does not divide 78.

Correct solutions were submitted by Peter Kosek and Elizabeth Mellen.

Problem for graduate students:

Prove that every power of 13 can be written as sum of two squares. That is, for every $n \geq 1$ there are two integers a_n and b_n such that $13^n = a_n^2 + b_n^2$.

Last week's problem:

Solve the equation

$$2(x^2 - x + 3) + 2(y^2 - y + 1) = 7$$

$$\iff x^2 - x + 3 + y^2 - y + 1 = \frac{7}{2} \iff x^2 - x + y^2 - y + \frac{1}{2} = 0$$

$$\iff x^2 - x + \frac{1}{4} + y^2 - y + \frac{1}{4} = 0 \iff \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = 0$$

$$\iff x = y = \frac{1}{2}.$$

Correct solutions were submitted by Darryl George, Ellen Nary and Kevin Spear.