

In a pond there are 128 amphibians, either tadpole or frog. Tadpoles have a tail, while frogs don't. All frogs have four legs, while tadpoles, according to how old they are, can have four legs, two legs, or none.

In the pond there are 264 legs and 113 tails and one kind of tadpole is twice as numerous as another.

How many two - legged tadpoles are there in the pond?

Deadline for solution: 09/28/09. Send solution at gprajitu@brockport.edu or drop hard copy in Dr. Prajitura's mail box.

Last week's problem:
If we compute the sum

$$1 + 11 + 111 + 1111 + \dots + 11\dots 1,$$

where the last number has 96 digits, how many 1's will be in the result?

The answer is 11. One of the ways to solve the problem involves the formulae:

$$1 + x + x^2 + \dots + x^n = \frac{x^{n+1} - 1}{x - 1},$$

which is the sum of a geometric progression, and

$$1 + 2x + 3x^2 + \dots + nx^{n-1} = \frac{nx^{n+1} - (n+1)x^n + 1}{(x-1)^2},$$

which is the derivative of the first one.

The sum can be seen as

$$\begin{aligned} & 96 + 95 \cdot 10 + 94 \cdot 10^2 + 93 \cdot 10^3 + \dots + 2 \cdot 10^{94} + 10^{95} \\ &= 96(1 + 10 + 10^2 + 10^3 + \dots + 10^{95}) - (10 + 2 \cdot 10^2 + 3 \cdot 10^3 + \dots + 95 \cdot 10^{95}) \\ &= 96 \frac{10^{96} - 1}{10 - 1} - 10(1 + 2 \cdot 10 + 3 \cdot 10^2 + \dots + 95 \cdot 10^{94}) \\ &= 96 \frac{10^{96} - 1}{9} - 10 \frac{9510^{96} - 9610^{95} + 1}{81} = 96 \frac{10^{96} - 1}{9} - \frac{10^{96}854 + 10}{81} = \frac{10^{97} - 874}{81}. \end{aligned}$$

Therefore we must divide the number 999...9216 (where 9 repeats 93 times) by 81 or to divide the number 111...1024 by 9. The division is periodic except

the last three times and the result has the group 123456790 repeating 10 times followed by 123446. Hence 1 shows 11 times.

The problem was solved by Peter Kosek, Joshua Swanson, Elizabeth Miller, and Sonja Larson.

Problem for graduate students:

Prove or find a counterexample: If (a_n) is a sequence such that for every $k \geq 2$, $\lim_n a_{kn}$ exists (and is the same for each k) then $\lim_n a_n$ exists.