Deadline is October 19.

Undergraduate problem:

An integer is called deficient if the sum of all its proper divisors (that is, all the divisors except for the number itself) is less than the number. For example all prime numbers are deficient. 10 is another example of a deficient number.

Prove that there are infinitely many deficient numbers having two prime factors.

Graduate problem:

For every $k \geq 3$ prove that there are infinitely many deficient numbers having k prime factors.

Last week's problems:

Undergraduate problem:

$$p^2 - q^2 = 2009 \iff (p-q)(p+q) = 2009.$$

Each way of factoring 2009 will give a solution. There are 3 such ways: $1 \cdot 2009$, $7 \cdot 287$, and $41 \cdot 49$. Notice that the first factor, $p - q$ must be less than the second, $p + q$.

The first way of factoring gives us $p - q = 1$ and $p + q = 2009$, from where we get $p = 1005$ and $q = 1004$. The other two cases can be solved in the same way.

Correct solutions were submitted by: Peter Kosek, Tsvetomira Radeva, Sonja Larson, Joshua Swanson, Elizabeth Mellen, and Logan Martin.

Graduate problem:

The statement to prove is: n is the difference of two squares if and only if n is not congruent 2 mod 4.

To prove this, we start as before:

$$p^2 - q^2 = n \iff (p-q)(p+q) = n.$$

Let $n = uv$ be a factoring of n with $u < v$. We get the system $p - q = u$ and $p + q = v$, which, adding and subtracting the two equations, leads to $2p = u + v$ and $2q = v - u$. Therefore the numbers $u + v$ and $v - u$ are both even which implies that u and v are either both even or both odd. If they are both odd then n is odd, if they are both even then n is divisible by 4.

So far I showed that if it is possible to write n as difference of two squares then n is not congruent 2 mod 4. To finish the problem we must show that in all other cases is possible to factor n as a product of either two odds or two evens. If n is odd than there is at least one such way, $1 \cdot n$. If n is a multiple of 4 then again there is at least one such way $2 \cdot \frac{n}{2}$.

Correct solutions we submitted by Douglas Bunker, Peter Kosek, Tsvetomira Radeva, Elizabeth Mellen, and partially, by Logan Martin.