Problem of the week

Week 6

Fall 2011

Undergraduate problem

Find

$$\lim_{x\to\infty} x^{2k}\left(\frac{\arctan x^k}{x^k} - \frac{\arctan(x^k+1)}{x^k+1}\right)$$

where k is a positive number.

Last problem:
Prove that

$$\sqrt{a^2+b^2+8a-6b+25}+\sqrt{a^2+b^2-4a+10b+29}\geq 10$$

$$\sqrt{a^2+b^2+8a-6b+25}+\sqrt{a^2+b^2-4a+10b+29}\geq 10$$
$$\iff \sqrt{(a+4)^2+(b-3)^2}+\sqrt{(a-2)^2+(b+5)^2}\geq 10$$

Let $P:(a,b); Q:(-4,3); R:(2,-5)$. The inequality says that $PQ+PR\geq QR$ which is true because the three segments are the sides of a triangle.

The problem was solved by Riley Bauer. Partial solutions were sent by Nicole Cannavino, Natalie Mundt and Gregory Barron.

Graduate problem

If $a, b \in [0,1]$, prove that

$$\frac{a^2+b^2}{1+a^2b^2} \leq (a+b-ab)^2.$$

Last problem:
Find all numbers n such that $n^2+59n+881$ is a square.

If $n^2+59n+881$ is a square then $4(n^2+59n+881) = 4n^2+236n+3524$ is also a square, say k^2.

$$k^2 = 4n^2+236n+3524 \iff k^2 = (2n+59)^2+43 \iff k^2-(2n+59)^2 = 43$$

$$\iff (k-2n-59)(k+2n+59) = 43.$$

43 is a prime number and can be factored only as $1 \cdot 43 = 43 \cdot 1 = (-1) \cdot (-43) = (-43) \cdot (-1)$. We get 4 system of equation and solving them we get only two solutions, $n = -19$ and $n = -40$.

The deadline to submit a solution is Friday November 18.

You can email the solution to Dr. Prajitura at gprajitu@brockport.edu or bring a hard copy to Brown 296. Don't forget to put your name on your submission.